Plate and Gap Acoustic Waves for Highly Sensitive Gas and Liquid Sensors

M.Yu. Dvoesherstov
Radio-physical department
State University
Nizhny Novgorod, Russia
dvoesh@rf.unn.ru

V.I. Cherednick
Chemical department
State University
Nizhny Novgorod, Russia
odissey@sandy.ru

K.Bhattacharjee
Clarisay
Dallas, USA

***Abstract*—Properties of acoustic waves in thin plates and of gap waves are investigated theoretically and search of optimal orientations in piezoelectric plates is performed for highly sensitive gas and liquid sensor(s) designs.**

Keywords-plate waves; gap waves; optimal orientations; gas sensors; liquid sensors

I. INTRODUCTION

One of the ways of improvement of performance of acoustic liquid and gas sensors is the use of nontraditional types of acoustic waves. Waves in thin piezoelectric plates with thickness comparable to acoustic wavelength λ, can be used for this purpose. Acoustic sensor may be developed by using gap waves, propagating along the gap between two piezoelectric plates. Liquid or gas can be directed along this gap too.

The aim of this work is theoretical search of orientations in various piezoelectric plates or in the system of two plates with the gap between them with propagation characteristics that are optimal from the point of view of design of high sensitive acoustic liquid and gas sensor(s).

II. ACOUSTIC PLATE WAVES

Depending on symmetry group of a crystal and the wave propagation direction, waves of two types may exist: Lamb waves and shear horizontal waves (SH-waves). The well known Campbell-Jones technique [1] is used for analysis of properties of acoustic waves in the thin piezoelectric plate of any crystal symmetry group. The general solution for mechanical displacements u_i and for electric potential φ is the sum of 8 partial waves:

$$u_i = \sum_{n=1}^{N} A_n \alpha_i^{(n)} \exp(i\kappa\beta^{(n)} x_3) \exp\{i\kappa[x_1 - Vt]\} \qquad (1)$$

$$\varphi = \sum_{n=1}^{N} A_n \alpha_4^{(n)} \exp(i\kappa\beta^{(n)} x_3) \exp\{i\kappa[x_1 - Vt]\}.$$

Here $\alpha_i^{(n)}$ (i = 1-3), A_n – amplitude coefficients, $\beta^{(n)}$ – decrement coefficients along the x_3 axis, which is normal to the surface, x_1 – the wave propagation direction, κ - wave number, V – velocity, t – time, and N = 8.

Substitution of general solution (1) into coupled equations of the piezoelectric medium gives the Christoffel equation in the matrix form [1]. Depending on crystal symmetry conditions and on the wave propagation direction, the Christoffel equation may be degenerate, i.e. may have two independent solutions. When the elastic constants C_{ij} and the piezoelectric constants e_{ij} satisfy the following conditions:

$$C_{14} = C_{16} = C_{34} = C_{36} = C_{45} = C_{56} = 0; \quad e_{14} = e_{16} = e_{34} = e_{36} = 0 \qquad (2)$$

one of the independent solutions is a Lamb wave of vertical-longitudinal polarization with two components of mechanical displacements u_1 and u_3 and the electric potential φ.
In case of the following conditions getting satisfied:

$$C_{14} = C_{16} = C_{34} = C_{36} = C_{45} = C_{56} = 0; \quad e_{11} = e_{13} = e_{15} = e_{31} = e_{33} = e_{35} = 0 \qquad (3)$$

one of the independent solutions is a shear horizontal wave with one transversal component of mechanical displacement u_2 and the electric potential φ.

In general case of crystal symmetry the Christoffel equation gives the single solution and all the modes in the plate have all three components of mechanical displacements u_1, u_2, u_3 and the electric potential φ.

For further calculations, 8 boundary conditions on upper "1" and lower "2" boundaries of the plate are needed:

$$T_{3i}^{1,2} = 0, \; D_3^{1,2} = D_3^{f} \qquad (4)$$

Here T_{3i} is tensor of normal stresses, D_3 is the electric displacement, i = 1-3.

Phase velocities V_k of all the plate modes may be found from boundary equations.

Many plate modes with different phase velocities exist, therefore, one must select rather narrow velocity interval for searching of every such mode. Visualization of the boundary condition function in the work program window was used for reliable definition of every such interval.

Usually electromechanical coupling coefficient K^2 for surface acoustic waves is calculated as [1]:

$$K^2 = 2(V_0 - V_S)/V_0, \quad (5)$$

where V_0 and V_S are wave phase velocities on open and short-circuited surfaces. For semi-infinite medium this is a single value for a given orientation. One can calculate the K^2 value in the thin plate by this formula too. This value depends on the plate thickness H for every mode. It must also be taken into account that the wave velocity in the plate depends on electric boundary conditions on both boundaries of the plate, i.e. both V_0 and V_S for the upper boundary depend on the conditions on the lower boundary (open or short-circuited). Therefore two different values of K^2 can be obtained for upper surface. Moreover, one can take V_0 for both open surfaces and V_S for both short-circuited surfaces. It gives the third K^2 value.

Figs. 1a and 1b show three variants of dependences of calculated K^2 on normalized plate thickness H/λ for zero symmetric (S_0 – Fig. 1a) and anti-symmetric (A_0 – Fig. 1b) Lamb modes for YZ-$LiNbO_3$ (1 – lower surface is open, 2 – lower surface is short-circuited, 3 – both surfaces are or open or short-circuited).

Dependences of the phase velocities on the thickness are presented in Figs. 1a and 1b too (curve 4).

Dependences of K^2 are not monotonic, whereas both velocities monotonically approach to the velocity of surface acoustic wave in semi-infinite medium (V_{SAW} = 3.487 km/s) for H/λ > 3, one from zero value, another – from a value which is approximately equal to the velocity of the longitudinal bulk wave.

The values of K^2 also approach to the value corresponding to SAW ($K^2_{SAW} \approx 4.38$ %), but not in all the cases.

If lower surface is open, then K^2 of low-velocity mode approaches to K^2_{SAW} (curve 1 in Fig. 1a), whereas K^2 of high-velocity mode decreases to zero (curve 1 in Fig. 1b). If lower surface is short-circuited, the contrary happens (curves 2 in Figs. 1a and 1b).

For the third variant of K^2 (both surfaces are open or short-circuited) this value approaches to K^2_{SAW} for both zero modes.

Moreover, the condition $K_3^2 = K_1^2 + K_2^2$ is satisfied for any plate thickness for both zero modes (indexes 1, 2, 3 correspond to three mentioned variants of K^2).

Fig. 2 shows calculated dependences of normalized mechanical displacement magnitudes u_1 and u_3 along the plate thickness (H/λ = 0.5) for high-velocity zero mode on YZ-$LiNbO_3$. Distribution of u_1 along the plate thickness is symmetric, distribution of u_3 is anti-symmetric (absolute value of u_3 is shown in Fig. 2).

If condition (3) is satisfied, then SH-wave without u_3 displacement propagates in the plate.

It means that such wave propagates without radiation loss, if contact of liquid and the plate takes place. Moreover K^2 of SH-wave may be very large for some thicknesses [2-3]. Fig. 3 shows calculated dependences of velocities (dotted lines) and K^2 (solid lines) of SH-waves on normalized plate thickness H/λ for XY-$LiNbO_3$, YX-$KNbO_3$, YX-PKN and XY-$LiTaO_3$ (curves 1, 2, 3, 4 respectively). Velocities are calculated under condition, that both surfaces are open, and values of K^2 are calculated under condition, that lower surface is open. Material constants are taken from works [4-6].

As one can see in Fig. 3, maximal value of K^2 is: for XY-$LiNbO_3$ - about 35 % (H/λ = 0.06, V = 4.35 km/s), for YX-$KNbO_3$ – about 99.3 % (H/λ = 0.12, V = 4.67 km/s), for YX-PKN – about 54.5 % (H/λ = 0.02, V = 3.04 km/s) and for $LiTaO_3$ – about 10.7% (H/λ = 0.06, V = 3.75 km/s).

Such extremely high values of K^2 in these crystals open the possibilities of effective control of the SH-wave velocity, for example by conductive screen near the crystal surface [7].

This feature can also be utilized for highly sensitive acoustic-electronic liquid and gas sensors design [8].

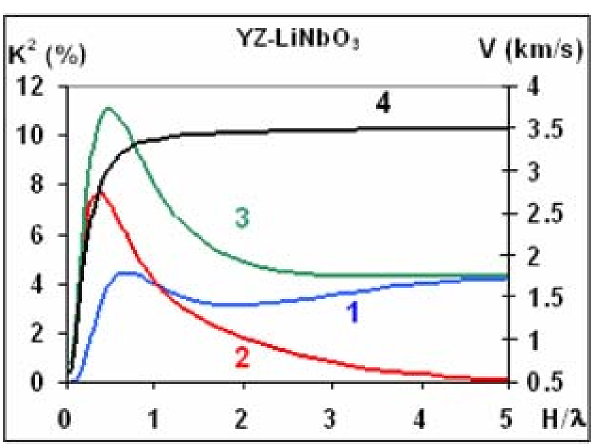

Fig. 1a. Dependences of K^2 (curves 1, 2, 3) and velocity (curve 4) on normalized plate thickness. Zero symmetric mode.

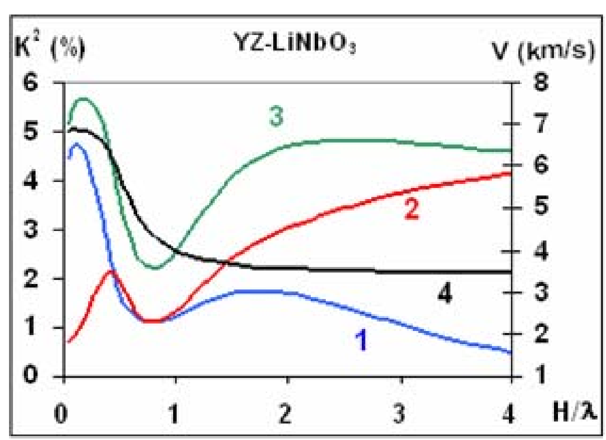

Fig. 1b. Dependences of K^2 (curves 1, 2, 3) and velocity (curve 4) on normalized plate thickness. Zero anti-symmetric mode.

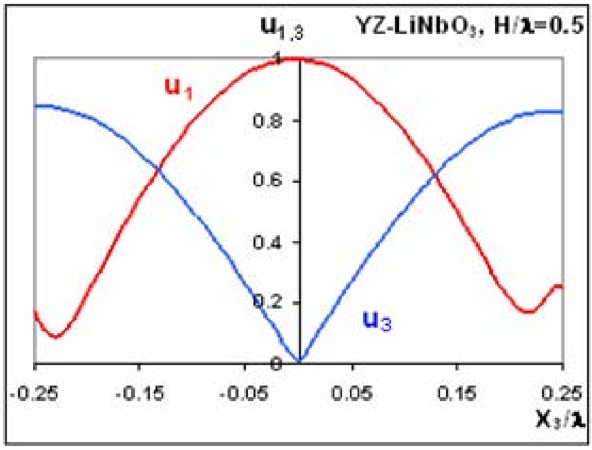

Fig. 2. Distribution of mechanical displacements along the plate thickness.

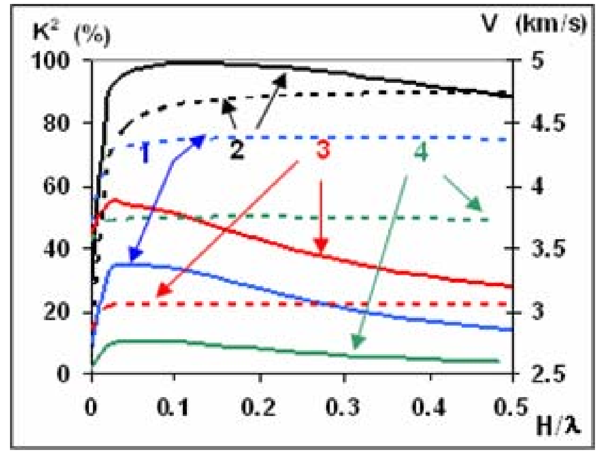

Fig. 3. Dependences of velocity (dotted lines) and K^2 (solid lines) on normalized plate thickness.

III. Acoustic Gap Waves

Gap wave propagates along the gap between two piezoelectric media. The electric power is localized inside the gap and maxima of the mechanical displacements are localized near surfaces and decrease exponentially into the depth of the media.

Mechanical displacements u_i in both piezoelectric media are coupled with each other by the electric field of the wave. The velocity of the gap wave depends on the wavelength λ because of presence of the structure with finite size (the gap with width H). Such space dispersion is analogous to dispersion in wave guides.

The general solution can be obtained from coupled equations of both piezoelectric media (1 and 2) together with 10 boundary conditions:

$$T_{3i}^{1} = 0\,,\ T_{3i}^{2} = 0\,,\ \varphi^{w} = \varphi^{1},\quad D_3^{1} = D_3^{w},\qquad (6)$$
$$\varphi^{w} = \varphi^{2},\quad D_3^{2} = D_3^{w}\,,\ \text{and } i = 1\text{-}3$$

In general case, displacements $u_i^{1,2}$ and potential $\varphi^{1,2}$ in each piezoelectric medium can be written as sum of four partial waves (1), where N = 4. The electric potential φ^w inside the gap is determined by solving Laplace equation and may be presented in the following way:

$$\varphi^{w} = (\Phi_s{*}ch(kX_3) + \Phi_a{*}sh(kX_3)){*}\exp(jk(X_1-Vt))$$

Here, "ch" and "sh" are cosine and sine hyperbolic functions. Unknown coefficients of all the linear combinations, including Φ_s, Φ_a, can be determined from 10 complex homogeneous equations, which are given by boundary conditions (6). Number of unknown coefficients and equations can be reduced to 8 if coefficients Φ_s, Φ_a are expressed by potentials on both surfaces beforehand. Both methods of definition of linear combination of coefficients are equivalent.

Several variants of K^2 definition are possible because two surfaces exist in the considered system. In all the cases, the K^2 value rapidly decreases when gap width increases.

Gap wave can also exist in the structure: semi-infinite medium – gap – thin plate (thickness H ≈ λ). Technique described above can be used for obtaining solution in this case too, but solution in the plate must be presented as sum of 8 partial waves. It is necessary to add also four boundary conditions on the upper surface of the plate, namely zero values of normal components of mechanical displacements and continuity of the normal component of the electric displacement. The system of 14 boundary condition equations must be solved in this case (or 12, if coefficients Φ_s and Φ_a are determined beforehand).

Gap waves can also propagate in the system of two thin plates with the gap between them. The general solution in each plate can be obtained in this case as a sum of 8 partial waves. Total quantity of partial waves in both plates is 16 and Φ_s, Φ_a values must be determined too. Thus 18 boundary condition equations are necessary in this case (or 16, if coefficients Φ_s and Φ_a are determined beforehand).

SH-mode with very high K^2 can propagate in the thin plate (see Fig. 3). Therefore one can assume that SH-modes with high K^2 can exist also in the system of two plates with a gap between them. Fig. 4 shows calculated dependences of V_0 (dotted lines) and K^2 (solid lines) on the normalized gap width H/λ for three different values of normalized plate thickness $H_1/\lambda = H_2/\lambda = 0.1, 0.2, 0.5$ (H_1 – thickness of lower plate, H_2 – thickness of upper plate).

Plates are XY-$LiNbO_3$. Electromechanical coupling coefficient K^2 is calculated under condition that IDTs are placed on the upper surface of the upper plate and all the other three surfaces are open.

One can see in Fig. 4 that maximal value of K^2 for these dependences corresponds to both plate thickness $H_1/\lambda = H_2/\lambda = 0.1$. In particular, $K^2 \approx$ 9 % for gap width H/λ = 0.01.

In the system of two plates separated by a gap, many acoustic modes can exist as in the single plate.

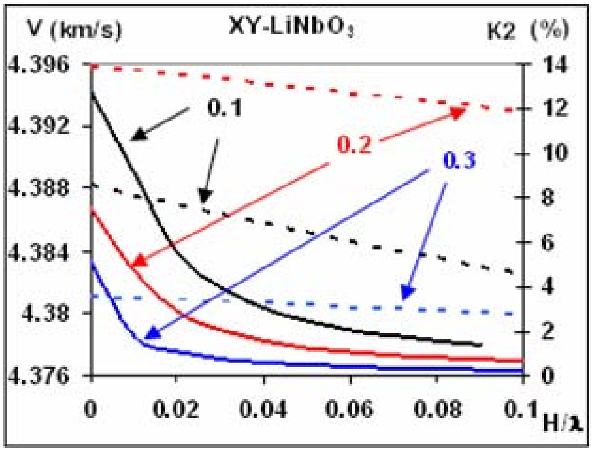

Fig. 4. Dependences of velocity (dotted lines) and K^2 (solid lines) on normalized gap width.

For example, for conditions corresponding to Fig. 4 for $H_1/\lambda = H_2/\lambda = 0.1$ and $H/\lambda = 0.01$ low-velocity mode with the velocity $V_0 = 4.04727$ km/s and $K^2 \approx 20.7\%$ and also high-velocity mode with $V_0 = 6.4753$ km/s and $K^2 \approx 4.83$ % exist.

So, SH-modes of gap waves between two plates also can be used for liquid and gas sensors designs.

IV. Conclusion

Plate and gap waves in some crystals are considered. It is shown, that such waves with very high electromechanical coupling coefficient may exist in some concrete plates and systems of two plates with a gap between them. These plates and systems of two plates can be used for highly sensitive liquid and gas sensor designs.

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